1   A number of the form $1/N$, where N is an integer greater than 1, is called a *unit fraction*.

Noting that

$$\frac{1}{2} = \frac{1}{3} + \frac{1}{6} \quad \text{and} \quad \frac{1}{3} = \frac{1}{4} + \frac{1}{12},$$

guess a general result of the form

$$\frac{1}{N} = \frac{1}{a} + \frac{1}{b} \tag{*}$$

and hence prove that any unit fraction can be expressed as the sum of two distinct unit fractions.

By writing (*) in the form

$$(a - N)(b - N) = N^2$$

and by considering the factors of N^2, show that if N is prime, then there is only one way of expressing $1/N$ as the sum of two distinct unit fractions.

Prove similarly that any fraction of the form $2/N$, where N is prime number greater than 2, can be expressed uniquely as the sum of two distinct unit fractions.

Please provide answers to the following discussion questions. Don't include full calculations in your responses, just explore the question and try to anticipate routes through it.

1. How would you go about guessing a general result from the examples? Is there anything you can do to give you further confidence in your guessed general result before you go on to prove it?

2. How could you prove your guessed general result? How do you start with this? How do you anticipate the algebra proceeding? What sort of care will be needed with implications (if any)?

3. How does (*) rearrange to this new form? What is the relevance of N prime in this question? If you can't see this straight away what exploration could you do to help with this?

4. How can you get started with the last part of the question? How might what you have done so far help you with 2/N?

Submit your answers by e-mail to stepeasterschool@maths.org by Friday 16th March 2012 with the subject line: Lecture 3 Question 1